

Flux Compactification of Type IIB Supergravity

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and gauged supergravity
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Introduction

Major problem in string compactifications:

emergence of moduli space of string vacua

moduli appear in two guises:

- (i) closed string moduli: deformations of internal cycles
- (ii) open string moduli: wrapped branes not rigid

standard model of particle physics & (inflationary) cosmology

⇒ These moduli have to be fixed !

If supersymmetry is broken only at fairly low energies

need fixing mechanism that preserves
some supersymmetry

So far : only fluxes seem to provide mechanism
for lifting *both* moduli while preserving some susy

fluxes = field strength of RR and NS forms
that are non-zero in the vacuum

(i) fluxes: expand (shrink) parallel (perpendicular) cycles

→ fix closed string (geometrical) moduli

But often: run-away behavior

(ii) they couple to Born-Infeld action → generate potential

→ can also fix open string moduli

they are compact - potential has always extrema

But also : metric fluxes (twisted tori e.g.)

all fluxes :

related to generalized Scherk-Schwarz reductions

these reductions are especially interesting, because

Scherk-Schwarz reductions yield consistent truncation
on massless KK spectrum

In addition: → related to gauged supergravity
⇒ can calculate the potential explicitly

and hence: - KK spectrum
- number of vacua
- de Sitter extrema

(→ section 2)

However: gauged supergravity does not yield the deformed (internal) geometry

⇒ lift to 10 dimension in most cases not clear !

To get new (internal) geometry

solve directly the 10-d equations

→ uncovers embedding of the fluxes

→ solution is classified by torsion classes

or the set of globally defined differential forms (G-structures)

(→ section 3)

remark: branes are charged ⇒ fluxes give embedding of branes so that back reaction of the branes on geometry is under control

branes vs. fluxes? - branes = δ -function sources in eom or Bianchi identity
- fluxes “near horizon” limit of branes

unfortunately: explicit solutions available only for special cases
e.g. in massive IIA – nearly Kahler spaces

(in section 4 → present a solution in type IIB supergravity)

after finding the spaces:

ask (the mathematicians) for moduli spaces
related to this geometry

Fluxes, Scherk-Schwarz reduction and gauged supergravity

Constraints on vacuum:

strong: 10-d metric flat and all fields are trivial

weak: only 4-d vacuum is Poincaré invariant

→ internal metric and RR/NS-fields can be non-trivial !

→ related by supersymmetry (back reaction of fluxes on geometry)

hence: internal metric and gauge potential are not constant !

BUT: in order to integrate over the internal coordinates,
gauge invariant quantities (Ricci tensor, field strengths)
have to independent on internal coordinates!

(generalized) Scherk-Schwarz reduction

Consider type IIB supergravity

bosonic fields :

common sector (e, B, ϕ)

RR-sector (C_0, C_2, C_4^+)

with the field strengths :

$$G_3 = \frac{F_3 - T F_3^*}{\sqrt{1 - |T|^2}}, \quad F_5 = dC_4 - \frac{1}{8}(C_2 \wedge dB)$$

$$dG_3 = -(P - iQ) \wedge G_3, \quad dF_5 = -\frac{1}{8} dC_2 \wedge B$$

$$T = \frac{1 + i\tau}{1 - i\tau}, \quad \tau = C_0 + i e^\phi, \quad P = \frac{dT}{1 - |T|^2}, \quad Q = \frac{\text{Im}(T dT^*)}{1 - |T|^2}$$

example: axion/dilaton coupling in type IIB supergravity

$$S \sim \int \sqrt{g} \left[R - \frac{g^{MN} \partial_M \tau \partial_N \bar{\tau}}{|\tau - \bar{\tau}|^2} \right] \quad \text{symmetry : } \boxed{\tau \rightarrow \tau + c}$$

Scherk-Schwarz ansatz : $\tau(x, y) = \tau(x) + m y$

metric ansatz : $ds^2 = e^{2\sigma} (dy + A_\mu dx^\mu)^2 + g_{\mu\nu} dx^\mu dx^\nu$

Langrangian : independent of internal coordinate y

$$\frac{g^{MN} \partial_M \tau \partial_N \bar{\tau}}{|\tau - \bar{\tau}|^2} = \frac{g^{\mu\nu} D_\mu \tau(x) D_\nu \bar{\tau}(x)}{|\tau(x) - \bar{\tau}(x)|^2} + \frac{m^2}{|\tau(x) - \bar{\tau}(x)|^2} e^{-2\sigma} \quad , \quad D_\mu \tau = \partial_\mu \tau(x) - m A_\mu$$

→ local symmetry :

$$\boxed{\tau \rightarrow \tau + c(x) \quad , \quad A \rightarrow A + \frac{dc}{m}}$$

\triangleq coordinate transformation of original metric

→ ie. corresponds to gauging of global shift symmetry

In general : any global symmetry can be used for
Scherk-Schwarz reduction

general procedure :

perform special (non-constant) symmetry transformation
so that, gauge invariant quantities remain independent
of internal coordinates (and can be integrated out)

result : charged scalar field + potential

hope: fixes all scalar fields

note: can gauge away the charged scalar

→ mass term for the gauge field

Bergshoeff, de Roo, Green, Papadopoulos, Townsend;
Myers, Kaloper; Dabholkar, Hull; (Scherk, Schwarz)

In the example before: flux related to 1-form $d_y \tau = m dy$

Same mechanism can be applied to other forms !

(a CY-compactification e.g., has no 1-forms or isometries)

$$B + iJ = u^a(x, y) \omega_a = [u^a(x) + c^a(y)] \omega_a$$

So that : $d(B + iJ) = d u^a(x) \wedge \omega_a + (m^k \chi_k + cc) = du \wedge \omega + H^{flux}$

with this H-flux : $F_5 = dC_4 - \frac{1}{8} C_2 \wedge H = (dA - \frac{1}{8} m C_2^{ext}) \wedge \chi + cc + \dots$

massive 2-form in 4 dimensions
~ dual to massive vector

potential comes now from $H_3^2 = G^{ab} m_a m_b = |m|^2$

Louis, Micu ; Dall'Agata

relation to gauged supergravity?

have to identify the Killing vectors

e.g. from the covariant derivative : $d q^u \rightarrow dq^u + k_I^u A^I$

gauged isometry : scalar dual to massive tensor $C_2^{(ext)}$

⇒ axionic shift universal hyper multiplet is gauged
→ straightforward to add other fluxes

result: each flux compactification certain gauging

But opposite statement is far from clear !

Relation to gauged supergravity rises the question:

Can we understand moduli stabilization within gauged sugra?

In order to generate a potential for all scalars

have to gauge isometries of M_H

(lets consider no gauging of M_V)

Killing prepotential : $P_I^x = K_{uv}^x \partial^u k_I^v$, K^x Kahler-2-forms

SU(2) valued superpotential : $W^x \sim X^I P_I^x$

symplectic section : $V = (X^I, F_I)$

Conditions for supersymmetric vacua :

fermionic variations have to vanish \Rightarrow

$$(i) \quad (\nabla_A X^I) P_I^x = 0 \quad , \quad (ii) \quad X^I (\nabla_u P_I^x) = K_{uv}^x (X^I k_I^v) = 0$$

Kahler cov. derivative

SU(2) cov. Derivative

note: we ignore here massive tensors – would give symplectic inv. expressions

Condition (ii): fixed point condition of Killing vector $k = X^I k_I$

complete lift of moduli: if fixed point set = NUT (point on M_H)

$$\Rightarrow \boxed{|k|=0 \quad , \quad \det(dk) \neq 0}$$

remark: if rank of 2-form dk is not maximal – fixed point set is extended
can parallel transport Killing vector along the fixed point set

note : the full quantum moduli space might have no isometries
→ fixing of moduli becomes unclear !

⇒ Condition (i): relation only for scalars in vector multiplets

namely: vector $P_I = P_I(q(z))$ has to be normal to M_V

→ can happen at different isolated points

(because $\nabla_A \nabla_B V \sim g_{AB} V$)

How about supersymmetric flat space vacua ?

requires additional input : $W^x = 0$

if the rotation : $(dk)_0 \in SP(2n)$

e.g. for homogeneous spaces, Killing vector must be outside of SU(2)-R-symmetry

On other hand if: $\det(dk) = 0$ potential has flat direction

→ no-scale structure is example of flat space vacuum

Deformed geometry and G-structures

Gauged supergravity approach:

Pro: obtain explicit potential (→ flat directions?)
→ address the issues of non-susy vacua, stability

Con: we dont know the 10-d geometry/fluxes
→ Are there supersymmetric cycles?
→ Can we wrap branes?

Alternative: solve the equations directly in 10 dimensions !

→ Killing spinor equations

$$0 = \delta \psi_M = \left(D_M - \frac{i}{2} Q_M + \frac{i}{480} F \Gamma_M \right) \epsilon - \frac{1}{96} (G \Gamma_M + 6 G_M) \epsilon^*$$

$$0 = \delta \lambda = iP \epsilon^* - \frac{i}{24} G \epsilon$$

notation : $G^{(3)} = G_{MNP}^{(3)} \Gamma^{MNP}$ etc.

ansatz for the bosonic fields (arbitrary function on internal coordinates)

$$\begin{aligned} ds^2 &= e^{-2V(y)} (g_{\mu\nu} dx^\mu dx^\nu + h_{mn} dy^m dy^n) \\ P &= P_m dy^m, \quad G = G_{mnp} dy^m \wedge dy^n \wedge dy^p \\ F &= 5 e^{-4V} \text{vol}_4 \wedge dZ + \text{Hodge-dual} \end{aligned}$$

external space : AdS $\Lambda < 0$ or Minkowski $\Lambda = 0$

ansatz for the fermions : $\epsilon = a \theta \otimes \eta + b^* \theta^* \otimes \eta^*$

With a single internal spinor – can define SU(3) structure

$$\eta \gamma_{mn} \eta^* = i e^{2\alpha} J_{mn}, \quad \eta \gamma_{mnp} \eta = i e^{2(\alpha+i\beta)} \Omega_{mnp}$$

Note: Killing spinor is singlet under structure group

Fluxes ~ deformation described by torsion classes

$$\delta \psi_m = 0 \quad \Rightarrow \quad \left(\nabla_m - \frac{1}{4} \tau_m^{pq} \gamma_{pq} \right) \eta = 0$$

intrinsic torsion components : $6 \times 15 = 6 \times (1+6+8)$
 SU(3) singlet spinor

$$42 \rightarrow (1+1) + (8+8) + (6+6) + (3+3) + (3+3) = W_1 + W_2 + W_3 + W_4 + W_5$$

scalar (1,1) (2,1) vector vector SU(3)- structures
 form form

(Chiossi, Salamon)

$$dJ = \frac{3i}{4} (W_1 \Omega^* + W_1^* \Omega) + W_3 + J \wedge W_4$$

$$d\Omega = W_1 J \wedge J + J \wedge W_2 + \Omega \wedge W_5$$

non-complex : $\tau \in W_1$ (nearly Kahler)
 $\tau \in W_2$ (almost Kahler)

complex : $\tau \in W_3$ (special hermitian)
 $\tau \in W_5$ (Kahler)
 $\tau = 0$ (Calabi Yau)

Fluxes in type IIB supergravity

Solving the Killing spinor equations:

external part
internal part

$$\delta \psi_\mu = 0 \quad , \quad \delta \lambda = 0$$
$$\delta \psi_m = 0$$

constraints on fluxes
differential eq. for spinor

A-type vacuum

$$\epsilon = a \left(\theta \otimes \eta + \theta^* \otimes \eta^* \right)$$

(Majorana spinor)

results: - only NS flux, all RR fields vanish
- common sector e.g. NS5-brane solution

B-type vacuum

$$\epsilon = a \theta \otimes \eta$$

results: - holomorphic axion/dilaton e.g. D3 and D7-branes
- internal space is still Kaehler or Calabi-Yau.

General case: $\epsilon = a \theta \otimes \eta + b^* \theta^* \otimes \eta^*$

results:

$$W_1 = W_2 = 0 \quad , \quad \Lambda = 0$$

$$b + i a = e^{-V/2 + i\alpha}$$

- internal space: complex (can use compl. coordinates z^i)
- external space: flat Minkowski

3-form:

$$G = \frac{1}{4} e^{-2V} J \wedge \left(\cot \alpha P_i dz^i + \tan \alpha P_{\bar{i}} dz^{\bar{i}} \right) + G^{prim}$$

$$\text{with} \quad J \wedge G^{prim} = \Omega \wedge G^{prim} = \bar{\Omega} \wedge G^{prim} = 0$$

remaining fields can be written as :

$$Q = d\theta - \frac{d\tau}{2\tau_2} \quad , \quad P = i e^{2i\theta} \frac{d\tau}{2\tau_2} \quad , \quad G_3 = i \frac{e^{i\theta}}{\sqrt{\tau_2}} (dA_2 - \tau dB_2)$$

$$F_5 \sim dZ \wedge vol_4 + \text{Hodge dual}$$

with (τ, θ, Z) given by

$$\tau = c_0 + i e^{\phi_0} \frac{|f|^2 \cos 2\alpha}{f \sin^2 \alpha + \bar{f} \cos^2 \alpha}$$

$$e^{-4V} = \frac{\operatorname{Re} f \sin^2 2\alpha}{4|f|^2 \cos 2\alpha}$$

$$Z = \frac{|f|^2 \cos^2 2\alpha}{\operatorname{Re} f \sin 2\alpha}$$

$$\tan \theta = -\frac{\operatorname{Im} f}{\operatorname{Re} f} \cos 2\alpha$$

where: $f = \frac{\tilde{Z}}{\cos 2\alpha + i \tan \theta} = f(z^i) \quad \tilde{Z} = Z \frac{\sin^2 2\alpha}{\cos 2\alpha}$

Remark: Supersymmetry leaves one function un-fixed !

→ fixed by equations of motion or Bianchi identities

here: α other possibility: Z can be fixed by $dF_5 = \Delta Z = \frac{5i}{12} G \wedge \bar{G}$

Discussion

$$f = \text{constant}$$

Apart from trivial solution, includes the flow type solution

→ fields depend only on one real coordinate

for $G \wedge \bar{G} = 0$ Z harmonic \Leftrightarrow is monoton (and singular!)

UV-regime \Leftrightarrow poles of Z (or warp factor)
 $\Leftrightarrow \alpha=0 \Leftrightarrow$ internal space becomes CY

geometry can be regular $\rightarrow AdS_5 \otimes S^5$

IR regime $Z \simeq 0$ or $\cos 2\alpha \simeq 0$

singular, NOT CY ! However, regular iff

$G \wedge \bar{G} \neq 0$ → internal space is always non-Kaehler
is necessary for flow to flat space

$$f \neq \text{constant}$$

Non-constant holomorphic function on compact space

\leftrightarrow has zeros and poles

zeros : $\sin 2\alpha \simeq 0$

- \rightarrow **B-type vacuum**: - Kaehler geometry (e.g. Calabi Yau)
- axion/dilaton is holomorphic
- prototype sugra solution: D3/D7-branes

poles : $\cos 2\alpha \simeq 0$

- \rightarrow **A-type vacuum**: - all RR fields are trivial
- prototype sugra solution: NS5-branes

Summary

to fix the moduli = one of the important issues in string theory
to get contact to particle physics and cosmology

fluxes ~ only possibility that preserves some supersymmetry (?)

two approaches : (i) **gauged supergravity**

→ gives explicit potential

but lift to 10 dimensions unclear

vacuum: fixed point of Killing vector

presented: criteria when all moduli lifted

(ii) **solving (Killing spinor) equations in D=10**

→ gives explicit geometry (defined by its torsion)

but there is no explicit potential (moduli spaces?)

presented: example on IIB side (complex spaces)

open questions: all moduli fixed, chiral matter ?

The End.

Moduli from Kaluza-Klein reduction

In low energy approximation:

type II string theory = type II supergravity

with N=2 supersymmetrie in D=10 supersymmetry

fermionic sector : two gravitinos and two dilatinos

opposite chirality \longrightarrow type IIA (non-chiral)

equal chirality \longrightarrow type IIB (chiral)

Lets assume we have a consistent truncation

then : **moduli = zero modes of KK scalar fields**

Scalars come from internal metric and RR/NS forms.

And moduli are in one-to-one correspondence to harmonic diff. forms of internal space.

Example: if $\omega_n \neq d \omega_{n-1}$, $d \omega_n = d^* \omega_n = 0$

→ get scalar field from $C_n = \phi(x) \omega_n + \dots$

because $F = dC = 0$ for $\phi = \phi_0 = \text{constant}$

→ **one modulus** (consequence of gauge symmetry)

in the following denote :

complete basis of 3-forms : $\{ \chi^k, \tilde{\chi}_k \} \in H^{(3)}(Y)$

complete basis of 2-forms : $\{ \omega_a \} \in H^{(2)}(Y)$

on IIA side :

$$C_3 = u^k \chi_k + A^a \wedge \omega_a + cc \quad , \quad B + iJ = z^a \omega_a$$

gravi-photon : C_1

hyper multiplet : $\{ u^k, v^k \}$, $v^k \sim$ internal metric

univ. hyper multiplet : $\{ \phi, B^{ext}, C_3^{(3,0)}, C_3^{(0,3)} \}$

vector multiplet : $\{ A^a, z^a \}$

on IIB side :

$$C_4 = A^k \wedge \chi_k + cc \quad , \quad B + iJ = u^a \omega_a \quad , \quad * C_4 + iC_2 = v^a \omega_a$$

gravi-photon : A^0

hyper multiplet : $\{ u^k, v^k \}$

univ. hyper multiplet : $\{ \phi, C_0, C_2^{ext}, B^{ext} \}$

vector multiplet : $\{ A^a, z^a \}$, $z^a \sim$ internal metric

on CY: complex structure and Kahler class deformation independent :

$$M = M_V \otimes M_H \quad \text{special Kahler} \otimes \text{quaternionic Kahler}$$

KK – approach : $M_{10} = M_4 \otimes M_6$, $x^M = \{ x^\mu, y^m \}$
and integrate over y^m

in general : Fourier expansion in set of harmonic eigenfunctions

→ 4-d massive mode

simplest case : fields do not depend on y^m
= truncation on the massless sector

truncation possible : if massless field \neq sources for massive

ie. no linear couplings → consistent truncation

BUT : requires explicit metric (only few examples known Nicolai, deWit)

weaker version : if source terms contain derivatives
then integrating out massive modes
does not change Lagr. on 2-derivative level

Duff, Ferrara, Pope, Stelle

(only this form expected for KK-reductions related to gauged supergravity)